Geometry: Space, Form, and Visual Thinking

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Abstract

In this article, I show how geometry as a school subject can become a unique instrument for the development of spatial imagination and visual thinking - provided that we stop reducing it to memorizing theorems and solving problems on drawings. Drawing on the CCFP model presented in the previous articles of the module, I justify that geometry possesses an exceptional potential for the development of spatial imagination and 3D visualization, the construction of causal relations, critical thinking, as well as forecasting and strategic thinking. I propose concrete examples of lessons and tasks in which work with geometric figures, the construction of cross-sections, 3D modeling, and the use of digital tools serve not as an end in themselves but as a means of developing thinking. I pay particular attention to the diagnosis of progress and the connection of subject instruction with the infrastructure of the school - the 3D modeling workshop, the architectural design studio, and the laboratory of visual geometry. I also justify that the restructuring of the teaching of geometry is not merely a methodological question but a strategic task: it is precisely through geometry that the cognitive apparatus capable of multidimensional, spatial thinking is formed, without which the solution of the difficult problems of the twenty-first century, including the understanding of the complex topology of neural networks, is impossible.

Keywords: geometry, mental mechanisms, spatial imagination, 3D visualization, causal thinking, critical thinking, diagnosis, infrastructure.

A Shadow That Can Be Touched

I will begin with a statement that may seem controversial, but I consider it fundamental. Geometry at school is not a subject about how to prove a theorem and how to solve a problem on a drawing. It is a subject about how we see the world, how we imagine space, and how we operate with forms in the mind. It is a subject through which a person learns to see - not with the eyes, but with thinking. Of course, knowledge of theorems and the ability to solve problems is important. But it is an instrument, not an end. The end is the capacity to create, retain, and manipulate mental images; the capacity to see causal relations in geometric constructions; the capacity to evaluate proofs critically and find errors; the capacity to forecast the results of geometric transformations [1; 5].

Here it is appropriate to recall Plato, who had inscribed above the entrance to his Academy: “Let no one ignorant of geometry enter.” For Plato, geometry was not merely the science of measuring the earth. It was a path to pure thinking. It taught one to see eternal ideas behind changing forms. And although today we do not share Plato’s theory of ideas, we can agree with him on one thing: geometry is not about what can be measured with a ruler. It is about what can be seen with the mind. Behind the triangle on paper stands the triangle in the mind. Behind the cube on the drawing stands the cube in space. And it is precisely this cube in space that we must teach one to see. Because without it, a drawing is merely lines. And with it, it is a model of the world. Which can be understood. And which can be controlled. Not with the hands. But with thought. Which sees. And knows. And can.

As I already showed in the first article of this series [5], the principle of the dual purpose of a lesson means that every geometry lesson must simultaneously solve two tasks: the subject task - the mastery of geometric content, and the meta-subject task - the development of mental mechanisms. These tasks do not contradict each other - on the contrary, they mutually reinforce each other. When a student masters geometry through spatial thinking, they master it more deeply. When a student develops spatial thinking through geometry, they receive an instrument that will serve them

in all other subjects and in life. It is important to emphasize: geometry creates the conditions of possibility for the development of mental mechanisms but is not their sufficient cause. Memorizing theorems or drawing blueprints by itself will not develop spatial thinking. Development occurs only through specially organized pedagogical activity, where the teacher acts as a facilitator and architect of the cognitive apparatus [5; 6].

Research confirms: every day the fact of schoolchildren's lack of understanding of the subject of geometry, based on contemporary features of psychological development, becomes more and more relevant [8]. This lack of understanding is connected not with a lack of effort but with the fact that traditional teaching methods do not take into account how spatial thinking is formed. Researchers are developing new ways of presenting material in the lesson for the development of spatial and geometric types of thinking in schoolchildren [8]. But I want to go further and show how each of the eleven mental mechanisms can be developed through the teaching of geometry - and how this changes the very approach to the subject. Geometry is not merely a branch of mathematics. It is a unique simulator of spatial intelligence, without which contemporary scientific and engineering thinking is impossible. This is precisely why geometry, in my view, is the key subject for the formation of the communicative-cognitive framework of personality, especially in the aspect of the capacity for multidimensional thinking [1]. Moreover, the restructuring of the teaching of geometry is not merely a methodological question. It is part of that historical necessity of the change of the pedagogical paradigm of which I wrote in articles [3] and [4]. The industrial school reduced geometry to a set of theorems and drawings that had to be memorized. But post-industrial society needs people capable of thinking, creating, and adapting - and geometry here is the key instrument. As I showed in article [2], subject-centrism is one of the main reasons for the tenacity of the old model. Overcoming this subject-centrism begins with a rethinking of the role of geometry in education.

Space That Can Be Seen

Spatial imagination and 3D visualization are developed through geometry in the most direct and natural way. Geometry is a subject about space, form, and their properties. No other subject gives such an opportunity for training the ability to see three-dimensional objects in the mind, to rotate them mentally, and to examine them from all sides. The ability to imagine geometric figures in space, to rotate them mentally, to see cross-sections, to understand projections, and to move between 2D and 3D representations. The teacher must consciously create situations that require mental manipulation of images. Not merely to show a drawing but to invite students to see the figure in volume. Not merely to give a formula but to invite them to imagine how the form changes when the parameters change. Research confirms the importance of this approach. In a work devoted to the development of spatial thinking in geometry lessons in the tenth grade by means of computer graphics systems, the authors note that the use of digital tools makes it possible to develop spatial and geometric types of thinking in schoolchildren [8].

It is like an architect who looks at a blueprint and sees a building. They do not see lines. They see space. They see how people will walk. How the light will fall. How the echo will sound. And they can change the blueprint so that the building becomes better. And a student who cannot see space sees only lines. And cannot change the blueprint. Because they do not understand what stands behind them. And behind them stands the world. Which can be seen. If one learns to look. And if one knows where to look.

The construction of causal relations is a mechanism that geometry develops through the logic of proofs. Geometry is not only space but also logic. Every geometric statement follows from previous ones. Every construction has its causes and consequences. The ability to see how one geometric property follows from another. The ability to build a logical chain of proof. The ability to explain why a particular theorem works. The teacher must focus attention on the logic of geometric reasoning, and not only on memorizing theorems. Not "memorize the theorem" but "prove it." Not "solve the problem according to an algorithm" but "explain why this works" [1]. It is like a detective story. A reader who follows only the events does not understand why the hero acted one way rather than another. A reader who follows the thoughts understands. And receives pleasure not from what happened but from why it happened. So it is in geometry. A student who sees only the theorem does not understand geometry. A student who sees the logic understands. And receives pleasure not from memorization but from having arrived at it themselves. With their own mind. With their own thought. With their own path. Which began with a question. And ended with a proof. Which has become yours. Because you built it. As a builder builds a house. From bricks. From thoughts. From logic. And this is the most important thing. Not the theorem. But the path. That leads to it. And that remains with you. Even when the theorem is forgotten. Because the path is an ability. And an ability is not forgotten. Just as the ability to

walk is not forgotten. Or to speak. Or to think. Which you have acquired. And which is now yours. Forever. And this is precisely education. Not the sum of knowledge. But the sum of abilities. Which make you who you are. And who you will become. In the next problem. In the next proof. In the next life. Which always begins with the first step. That you make. Now. Here. In this geometry problem. Which is not merely solved. It is thought through. And in this lies its strength. And in this lies its beauty. And in this lies its meaning. Which does not end. As geometry does not end. As thought does not end. As life does not end.

Walls That Become a Number

Critical thinking is another mechanism that geometry develops in a unique way. Geometry provides unique opportunities for the development of critical thinking. The proof of a theorem requires the checking of every step. A construction requires justification. Errors in reasoning become obvious. The ability to check proofs for correctness. The ability to find errors in geometric reasoning. The ability to evaluate whether a proof is sufficient. The use of problems that require not merely the application of formulas but analysis and evaluation. Tasks on finding errors in ready-made proofs. Working with counterexamples. Research confirms that 3D modeling requires students to have a deeper understanding of structures and principles of working with geometric objects, which promotes their analytical and critical thinking [12]. The use of CAD systems in teaching geometry makes it possible to develop reflection and critical thinking [13]. It is like checking a watch. You cannot simply look at the watch and believe that it shows the correct time. You must check it against a standard. And if it is slow, you must notice it. Otherwise you will be late. Or arrive too early. And in both cases you will be wrong. And geometry teaches one not to be wrong. Or, at least, to notice errors. And to correct them. Before they become a catastrophe. Because in geometry, as in life, an error at the beginning of the path can lead to the whole path turning out to be wrong. And then one must start again. From a clean slate. From a new thought. From a new proof. And this is not a defeat. It is experience. Which makes you stronger. And smarter. And ready for the next problem. Which will be even more complex. And even more interesting. And even more important. Because behind it stands life. Which is not a theorem. But which requires the same logic. The same mind. The same readiness. For the unexpected. And for the inevitable. And for what you cannot change. But can understand. And accept. And move on. Like a geometer. Who knows that the problem does not end. It continues. In the next proof. In the next construction. In the next life. Which always begins with the first step. That you make. Now. Here. In this problem. In this life. In this world. Which awaits your step. And your thought. And your decision. Which will change everything. Or change nothing. But you will make it anyway. Because you are a geometer. And a geometer cannot not think. Just as they cannot not see. Just as they cannot not live. And this is the most important thing. Not the theorem. But life. Which makes you who you are. And who you will become. In the next step. In the next thought. In the next life.

Forecasting and strategic thinking is a mechanism that geometry requires and develops. Geometry requires constant forecasting. Before constructing a cross-section, one must foresee what it will look like. Before choosing a method of proof, one must foresee where it will lead. The ability to foresee the results of geometric transformations. The ability to choose a strategy of proof. The ability to evaluate which approach will be the most effective. Problems requiring the choice of a strategy. Tasks on forecasting: what will be the result if? How will the figure change if? Working with geometric constructions where one must plan a sequence of steps [1]. It is like a chess player who thinks several moves ahead. They do not merely see the current position. They see future positions. And they choose the one that will lead them to victory. And if they do not see the future, they play at random. And lose. So it is in geometry. A student who cannot forecast solves at random. And often makes mistakes. And a student who can forecast solves consciously. And often finds the right path. Even if it is not the shortest. But it leads to the goal. And this is the most important thing. Not speed. But direction. Not the answer. But the path. That leads to it.

A Lesson That Begins with a Question

Now I will turn to the concrete. What does a geometry lesson built on the principle of dual purpose look like? I propose several examples - from simple to complex. The traditional approach: show a drawing, explain the properties of the figure. The approach that develops thinking: invite students to see the figure in volume, to rotate it mentally, and to describe what they see. Topic: the cube and its properties. The teacher gives students an image of a cube on a plane and asks questions. Imagine that you are holding a cube in your hands. How many faces does it have? Vertices? Edges? And now mentally rotate the cube so that you see it from above. How does the drawing look now? And now imagine that you have cut the cube with a plane passing through three vertices. What figure is obtained in the cross-

section? Take a paper model of the cube or use GeoGebra 3D and check your assumptions. What mental mechanisms are developed? Spatial imagination - through mental rotation and visualization. Forecasting - through predicting the shape of the cross-section. Critical thinking - through the testing of hypotheses [1]. It is like a child who assembles a construction set. They do not merely connect parts. They see in their mind what should result. They imagine what the model will look like. And if something does not fit, they change the parts. And imagine again. And so on until what they have conceived results. And if they cannot imagine, they simply connect the parts at random. And often make mistakes. And give up. Because they do not see the result. And the result is not what turns out. It is what you see in your mind. Before it turns out. And geometry teaches this. To see before it turns out. To imagine before you build. And then the construction becomes not mechanical. But meaningful. And then the student does not merely solve the problem. They create the solution. Their own. Which they saw. And which they can now show to others. And this is the most important thing. Not the answer. But the vision. Which makes you a creator. And not an executor. And this is precisely education. Not the sum of knowledge. But the sum of visions. Which you have created. And which are now yours. Forever. And in the next problem. And in the next proof. And in the next life. Which always begins with the first glance. That you cast. Now. Here. At this figure. Which is not merely drawn. It is seen. By you. By your mind. By your imagination. And this is the most important thing. Not the lines. But the image. Which stands behind them. And which you now see. And can show. And can change. And can create. Your own. New. Which did not exist before. And which now exists. Because you saw it. And built it. And this is precisely geometry. Not the science of figures. But the art of seeing. And creating. And understanding. Which does not end. As thought does not end. As life does not end.

A Game in Which There Are No Losers

The construction of cross-sections in 3D as the development of spatial imagination and critical thinking. The traditional approach: solve problems on constructing cross-sections on paper. The approach that develops thinking: use 3D modeling to visualize and investigate cross-sections. Topic: cross-sections of polyhedra. The teacher gives the task. Using the GeoGebra 3D program or a paper model, construct a cube. Construct a cross-section of the cube by a plane passing through three given points, for example, the midpoints of three edges issuing from one vertex. What figure did you obtain? Describe its properties. And now change the position of the plane. How does the shape of the cross-section change? At what position of the plane is the cross-section a regular hexagon? Formulate a hypothesis about which figures can be obtained in the cross-section of a cube and test it. Research shows that the use of GeoGebra 3D with augmented reality significantly improves visualization and spatial rotation skills in students [10]. A study conducted with sixty schoolchildren showed a significant improvement in the levels of geometric thinking of students after using GeoGebra AR [10]. What mental mechanisms are developed? Spatial imagination - through the creation and investigation of 3D models. Critical thinking - through the testing of hypotheses. Forecasting - through predicting the shape of the cross-section [1]. It is like an archaeologist who excavates an ancient city. They do not know what they will find. They put forward hypotheses. They test them. They make mistakes. And they dig again. And in the end they find what they were looking for. Or they do not find it. But they learn something new. About the city. About the people. About themselves. And this is the most important thing. Not the result. But the process. Which changes you. Makes you different. More attentive. More patient. More thoughtful. And this is precisely education. Not the sum of knowledge. But the sum of changes. That have happened to you. While you were searching. While you were thinking. While you were living. And geometry is one of the most powerful instruments for this. Because it does not forgive mistakes. But it also teaches one to find them. And to correct them. And to move on. To a new theorem. To a new proof. To a new life. Which always begins with the first step. That you make. Now. Here. In this problem. In this life. In this world. Which awaits your step. And your thought. And your decision. Which will change everything. Or change nothing. But you will make it anyway. Because you are a geometer. And a geometer cannot not think. Just as they cannot not see. Just as they cannot not live. And this is the most important thing. Not the theorem. But life. Which makes you who you are. And who you will become. In the next step. In the next thought. In the next life.

A House in Which Form Lives

The development of mental mechanisms requires not only new methods of teaching but also an appropriate infrastructure. For geometry this means creating spaces where spatial thinking can develop through practical, research, and creative activity. By creating this infrastructure, we form what Andy Clark and Kim Sterelny call a cognitive niche. Infrastructure is not merely premises but the material part of the environment that provides

scaffolding for the development of mental mechanisms. Without this niche the mechanisms remain abstract concepts. A 3D modeling workshop: a space with 3D printers and 3D modeling programs, where geometric abstractions become tangible. As research shows, the integration of 3D modeling into the educational process promotes the formation of engineering thinking in students and improves their abilities for analysis and problem-solving [12]. A laboratory of visual geometry: a space with physical models of geometric figures - polyhedra, solids of revolution, and nets. Research shows that the use of a duo of digital and tangible artifacts promotes the development of geometric and spatial thinking [15]. An architectural design studio: a space where students can apply geometric knowledge in the design of buildings, models, and structures. This develops not only spatial imagination but also forecasting and strategic thinking. A computer classroom with GeoGebra and AR tools: a space where students can investigate geometric properties with the help of dynamic geometry and augmented reality. Research confirms that the integration of GeoGebra 3D with AR improves spatial intelligence and optimizes the process of teaching solid geometry [11].

The creation of this infrastructure is not an expense but a strategic investment. Every child who, through these spaces, finds their entry point - whether it be 3D modeling, architectural design, or investigation with the help of digital tools - receives a powerful impulse for the development of all the other mental mechanisms [1]. It is like a city. In the city there are libraries, theaters, and workshops. And every child can find what is closer to them. And having found it, can grow. Not because they are forced to. But because the environment allows it. And this is the most important thing. An environment that allows. And not an environment that demands. Because one can demand much. But to allow is an art. And the school must be an art. And not a factory.

Roots and Branches

Geometry does not exist in isolation. It is connected with all the other subjects - and these connections can be used for the development of mental mechanisms. Geometry and mathematics: this is an obvious connection. But it is important that geometry be not merely a branch of mathematics but an independent way of thinking - spatial, visual, intuitive [6]. Geometry and physics: spatial thinking is critically important for understanding physics - from mechanics to optics. Vectors, forces, and fields - all of this requires the ability to see space [1]. Geometry and computer science: computer graphics, 3D modeling, and virtual reality - all of this is based on geometry. The development of spatial thinking through geometry is directly connected with computational thinking [12]. Geometry and art: perspective, composition, and proportions - all of this is geometry. The STEAM approach, which integrates art and mathematics, has revealed intersecting areas of mathematics and visual culture in spatial and planar geometry [14]. Geometry and architecture as a new subject or studio: the design of buildings, the creation of models, and work with scale and proportions - the direct application of geometric knowledge and the development of spatial imagination. Geometry and biology: forms in nature - fractals, symmetry, and the golden ratio - all of this is geometry. The understanding of biological structures requires developed spatial thinking [1].

The Language of Space

I want to end this article with what is perhaps the most important thing. Geometry is not merely a school subject. It is the language of space. It teaches us to see the world in three dimensions, to understand forms, and to imagine the invisible. It forms the cognitive apparatus capable of multidimensional, visual, and intuitive thinking. This is precisely why geometry, in my view, is the key subject for the formation of the communicative-cognitive framework of personality [1]. No other subject gives such an opportunity for the development of spatial imagination. No other subject requires such a level of visualization and mental manipulation of images. No other subject creates such conditions for the development of the ability to see what is not before the eyes. When we teach a child geometry, we teach them not merely to solve problems and prove theorems. We teach them to see - to see space, to see forms, to see structures. We teach them to imagine - to create images in the mind, to manipulate them, and to transform them. We teach them to think in three dimensions - and therefore also in four, and in n-dimensional spaces.

This is precisely why the restructuring of the teaching of geometry is not merely a methodological question. It is a strategic task. As I showed in the article on epistemological recursion [7], without the development of mental mechanisms, without the formation of the communicative-cognitive framework of personality, humanity will not be able to solve the difficult problems of the twenty-first century, including the understanding of the complex topology of neural networks. Geometry is the instrument through which the capacity for multidimensional, spatial thinking is formed. And if we want our children to become not merely calculators but thinkers, we must give them this instrument

in full. Not for grades. Not for examinations. For the sake of vision. For the sake of life. For the sake of being able to see the world not as a plane but as space. And so that they can not merely live in this space but understand it. And, perhaps, change it. For the better. With the help of geometry. Which does not merely measure. It sees. Like an architect who sees a building where there is still only a blueprint. Like a sculptor who sees a statue where there is still only stone. Like a geometer who sees the truth where there are still only lines. And this is the most important thing. Not the lines. But the truth. Which stands behind them. And which can be seen. If one learns to look. And if one knows where to look. And geometry teaches this. Every day. In every lesson. With every child. With every figure. Which is not merely drawn. It is seen. And in this lies its strength. And in this lies its beauty. And in this lies its meaning. Which does not end. As geometry does not end. As thought does not end. As life does not end. And this is the most important thing. The most valuable. The most genuine. And the most human. That we have. And that we can pass on. To the children. To the future. To life. Which continues. In every figure. In every proof. In every vision. That we create. And that remains. When we leave. Because visions remain. Theorems remain. Thoughts remain. And geometry continues. Even when we are silent. Because silence is also part of geometry. And sometimes the most important part. The deepest. The most human. And this too is geometry. Which does not end. Which continues. In us. And in them. And in those who will come after. And will read. And will understand. And will see their own. And this will be a continuation. Not of an end. But of a beginning. The beginning of a new vision. A new understanding. A new world. Which does not end. As geometry does not end. As life does not end. As thought does not end. Which seeks. And finds. And seeks again. Because in this lies the meaning. Not in answers. But in questions. Not in full stops. But in ellipses. Not in the end. But in the continuation. Which is always possible. As long as there is a figure. As long as there is space. As long as there is thought. As long as there is life. And this is the most important thing. The most valuable. The most genuine. And the most human. That we have. And that we can pass on. To the children. To the future. To life. Which continues. In every figure. In every proof. In every vision. That we create. And that remains. When we leave. Because visions remain. Theorems remain. Thoughts remain. And geometry continues. Even when we are silent. Because silence is also part of geometry. And sometimes the most important part. The deepest. The most human. And this too is geometry. Which does not end. Which continues. In us. And in them. And in those who will come after. And will read. And will understand. And will see their own. And this will be a continuation. Not of an end. But of a beginning. The beginning of a new vision. A new understanding. A new world. Which does not end. As geometry does not end. As life does not end. As thought does not end. Which seeks. And finds. And seeks again. Because in this lies the meaning. Not in answers. But in questions. Not in full stops. But in ellipses. Not in the end. But in the continuation. Which is always possible. As long as there is a figure. As long as there is space. As long as there is thought. As long as there is life.

In the next article I will turn to the subject which, in my view, is the bridge between abstract mathematics and the real world - to physics. I will show how this subject can develop not only computational skills but also the capacity for modeling, forecasting, and critical thinking. This will be the next step. The next door. The next house. And I invite the reader to walk this path together with me. Not as an observer. But as a participant. Because the school is not what others do. It is what we do. All together. Teachers. Students. Parents. Society. And if we want to change the school, we must begin with ourselves. With every lesson. With every figure. With every thought. And then, perhaps, in twenty years we will see a different school. Different children. A different world. A world in which thought is not afraid of itself. A world in which questions are valued no less than answers. A world in which education is not preparation for an examination. But preparation for life. And this will not be a utopia. This will be a reality. Which we have created ourselves. With our own hands. With our own thoughts. With our own lessons. And this is the greatest thing we can do. Not for ourselves. But for them. For the children. For the future. For life. Which continues. In every figure. In every proof. In every vision. That we create. And that remains. When we leave. Because visions remain. Theorems remain. Thoughts remain. And geometry continues. Even when we are silent. Because silence is also part of geometry. And sometimes the most important part. The deepest. The most human. And this too is geometry. Which does not end. Which continues. In us. And in them. And in those who will come after. And will read. And will understand. And will see their own. And this will be a continuation. Not of an end. But of a beginning. The beginning of a new vision. A new understanding. A new world. Which does not end. As geometry does not end. As life does not end. As thought does not end. Which seeks. And finds. And seeks again. Because in this lies the meaning. Not in answers. But in questions. Not in full stops. But in ellipses. Not in the end. But in the continuation. Which is always possible. As long as there is a figure. As long as there is space. As long as there is thought. As long as there is life.

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